A PREVALENT BIAS IN EVALUATING LIFE ANNUITIES

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Abstract
The initiation of benefits versus postponement of benefits decision and the optimal age for starting Social Security benefits are subjects of many recent papers. By law, benefits are paid only to live beneficiaries. Thus, the anticipated future benefits should be weighted by the recipient’s survival probabilities—the probabilities that the recipient is alive when the benefits will be received. Many published papers, mainly by business schools’ professors, assume that benefits will be received “on average” throughout the recipient’s expected remaining lifetime and estimate the present value of Social Security benefits by discounting the cash flow through life expectancy. This paper shows that the preferred approach is to estimate the Actuarial Present Value (APV) which weights each future payment by the probability that it will be received. Based on survival probabilities and life expectancy tables that are compiled by the CDC the paper demonstrates that the present value through life expectancy approach overstates the APV by approximately 5-8 percent. Therefore, timing decisions that are not based on the APV are incorrect.

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JEL codes: D81, J26, H55.

1. Introduction
Almost all American workers are entitled to receive Social Security benefits when they retire, and this entitlement represents an important source of retirement income for most retirees. The Social Security entitlement is often called Social Security Wealth by economists, see for example Martin Feldstein (1976) who argued Social Security wealth must be added to fungible wealth when figuring out people’s Total Wealth. Social Security Benefits (SSB) may be initiated at any age between 62 and 70. Retirees who choose to initiate SSB at a younger age, all other things equal, will receive smaller benefits than those who postpone initiation of SSB to a later age. On the other hand, early initiators receive their reduced SSB for a longer period. The optimal timing for initiating SSB has been the subject of many recent papers. It is generally agreed that the timing of initiation versus postponement of benefits decision may have significant consequences, but there is less agreement on how to model the problem or measure its financial implications. All recent papers use present value (PV) calculation to compare alternative future cash flows but they differ in the way they account for the fact
that SSB are paid only to live recipients who have uncertain lifetime (see Docking et al. 2013 for a recent literature review).

SSB are *Whole-Life-Annuitities* – the benefits are paid monthly for as long as the beneficiary survives\(^1\). Since SSB are paid only to live recipients, each anticipated future payment should be weighted by the probability that the payment will be received. That is, future payments should be weighed the probability that the recipient or his/her spouse will still be alive at the beginning of the period in which a payment is due. Thus, the expected present value of any future payment is determined not only by its discount rate assumption and cash flow definitions but also by the time horizon over which payments are assumed to be received.

The correct method to calculate the present value of SSB is the *Actuarial Present Value* (APV), also known as the *Expected Present Value*. The seminal paper that uses APV in economics (for the general case of demand for annuities) is Yaari (1965), but the actuarial literature is much older (see e.g. de Witt 1670-1672 and Allen 1907). The APV is computed by multiplying each present value of a future payment by the probability that it will be received and all the products are then added up. Examples of papers using the APV are Munnell and Soto (2007), Coile et al. (2002), Friedman and Phillips (2008, 2010) and Friedman (2018).

When they calculate the SSW many professionals financial advisers and business school professors assume that annuity payments will be received “on average” throughout a beneficiary’s Expected Remaining Lifetime (ERL) and estimated the present value of an Annuity Certain with a time horizon that equals the ERL.

However, as Jordan (1967) stated, this is a “persistent misconception.” Jordan proved mathematically that the APV of a life-annuity at age \(x\) is, in fact, *smaller* than “the present value of a life annuity certain for a term equals to the life expectancy at age \(x\) (see Jordan 1967, p. 174). Examples of papers using this approximation are Fraser et al. (2000), McCormack and Perdue (2006), Spitzer (2006), Docking et al. (2013) and many others.

This paper explains why the APV method is the preferred method when compared to Annuity Certain with ERL as the time horizon (ACLE). Using survival probabilities and life expectancy tables that are compiled by the Center for Diseases Control (CDC) this paper demonstrates that the ACLE approach overstates the APV by approximately 5-8 percent.

The ACLE approach is even more problematic when the present value of a joint annuity evaluated. In the USA married couples can choose to receive Social Security benefits as two single individuals, as an individual and his/her spouse, or as a surviving spouse (if the other spouse is deceased). For the case of married couples, the value of each period’s benefits depends on the probability that the spouse A and spouse B are jointly alive, the probability that spouse A is alive and the spouse B is dead and the probability that spouse B is alive and spouse A is dead. Thus, the APV cannot simply be approximated by the sum of spouse A and

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\(^1\) Actuaries call an annuity “whole-life” when payments are made for as long as the beneficiary survives. That is, survival is a condition for receiving payment. Most readers of this *Review* are familiar with “annuity-certain” which has a fixed (certain) time horizon.
spouse B’s ACLE as was done, for example, by McCormack and Perdue op. cit. and Docking et al. op. cit.

It should be noted that the SSB of married couples is a joint-life annuity, with benefits received until the second spouse dies. Thus, the life expectancy that matters is the couple’s joint life expectancy, not the spouses’ life expectancies as individuals: For example, the ERL of a 66-year-old male is 16.9 and the ERL of a 66-year-old female is 19.5, but the joint life expectancy of a married couple (where both husband and wife are 66) is 25.3 years — one of them is likely to survive that long². Moreover, consistent with the formal definition of ERL, the probability that a person will outlive its life expectancy is 50% (that is, 50% of people die before their ERL and 50% die after their ERL). For a married couple, the probability that at least one member of a couple will outlive his/her life expectancy 75%.

This paper proceeds as follows: section 2 discusses the difference between the APV and ACLE methods for a single retiree and for married couples. Section 3 presents numerical examples. Summary and conclusions are presented in Section 4.

2. Estimating Expected Present Value

Our goal is to calculate the value of an income stream that is contingent on the recipient being alive. Consider the following example. Sally is trying to determine when she should start collecting Social Security. Her latest Social Security statement shows that if she initiates SSB at her Normal Retirement age, 66, her monthly benefit will be $1,000. If she initiates at age 62, her monthly benefit will be $750, and if she postpones initiation to age 70 her monthly benefit will be $1,320. She has enough income from other sources that she can wait the extra eight years if it would be more beneficial to her. What should Sally do? On the one hand, she receives a much higher benefit if she waits. On the other hand, she might die before or soon after she reaches age 70. She can use a financial calculator or an Excel spreadsheet and compute the present value of the three alternative streams of payments, but how should she take her mortality into account? This is explained below.

Because the available Life Tables provide survival probabilities only for integer ages, the author will assume that the benefits are paid once a year at the beginning of each year³. Let $a(x, r)$ denote the actuarial present value of $1 to be received each year for as long as the recipient is alive, where $x$ = the recipient’s current age and $r$ = real interest rate. Then $a(x, r)$ is calculated as

$$APV = a(x, r) = 1 \cdot \sum_{t=x}^{\infty} p(x, t) \cdot \nu^t$$

² Data for joint life expectancy is from IRS publication 590, Appendix C, Table II.

³ The annual $p(x, t)$, can be converted to monthly probabilities by assuming that people are dying at a constant monthly rate between year $t$ and year $t+1$. Increasing the granularity of the life table will make the calculations more cumbersome but will not affect the main results.
Where \( p(x, t) \) is the probability that an individual aged \( x \) will be alive at age \( t \), 
\[ v^t = \frac{1}{(1+r)^t} \] 
is the discount factor and \( \Omega \) = the upper limit of the life table (100), 
\( p(x, t) \) is defined by:

\[ p(x, t) = \prod_{t=0}^{\Omega} (1 - q_{x+t}) \]  \hspace{1cm} (2)
Where \( q_x \) is the probability of dying between age \( x \) and \( x+1 \), \( (q_x \) is obtained from the life tables, such as those in Appendix A).

Equation (1) is often approximated by Equation (3), the ACLE, by arguing that income will be received on average through the ERL.

\[ ACLE = v(x, r) = \sum_{t=x}^{E(T_x)} v^t \]  \hspace{1cm} (3)
In Equation (3) \( E(T_x) \) is the ERL and \( T_x \) is the remaining lifetime of an individual aged \( x \) and \( v^t = \frac{1}{(1+r)^t} \).

According to Bowers et al. (1986, pp. 149-150), Jordan (1967, p. 174) and Milevsky (2006, p. 116), the approximation of the APV by using Equation (3) will overstate the APV, that is, \( v((x, r)) > (a(x, r)) \). This fact is a corollary of Jensen’s Inequality, a well-known mathematical theorem.

The relationship between Equation (1) and Equation (3) can be seen if a second term (whose value is zero) is added to the right-hand side of Equation (3). The second sum in Equation (4) is redundant but is included for ease of exposition:

\[ v(x, r) = \sum_{t=x}^{E(T_x)} 1 \ast v^t + \sum_{t=E(T_x)}^{\Omega} 0 \ast v^t \]  \hspace{1cm} (4)
Comparing Equation (1) with Equation (4) one can see that the ACLE approximation requires two assumptions that are rarely stated:
- All the payments until age \( E(T_x) \) will be received with certainty, that is \( p(x, t) = 1 \) for \( t \) smaller or equal to \( E(T_x) \).
- No payments will be received past \( E(T_x) \); that is \( p(x, t) = 0 \) for \( t \) greater than \( E(T_x) \).

The magnitude of the overstatement (bias) is shown in Table 1, below.

3. Numerical Example

Table 1 contrasts the results of APV and the ACLE calculations for a 66 years old retiree whose Social Security benefits are $1,000 per month. Since the US Life Tables are tabulated only for integer years it is assumed in the calculations that benefits are received as a single payment of $12,000 per year. The APV and ACLE values shown in the table were computed for three assumed real interest rate, 1, 2 and 3 percent and life expectancy (ERL) of 66 years old males = 16.9, and ERL of 66 years old females = 19.5.
Table 1. Present Value of Social Security Benefits evaluated at Age 66

<table>
<thead>
<tr>
<th>Real Interest Rate</th>
<th>ERL</th>
<th>APV a(x,r)</th>
<th>ACLE v(x,r)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>MALE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r=1%</td>
<td>16.9</td>
<td>$198,747</td>
<td>$188,878</td>
<td>$9,869</td>
</tr>
<tr>
<td>r=2%</td>
<td></td>
<td>$183,502</td>
<td>$172,155</td>
<td>$11,347</td>
</tr>
<tr>
<td>r=3%</td>
<td></td>
<td>$169,993</td>
<td>$157,810</td>
<td>$12,183</td>
</tr>
<tr>
<td>FEMALE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r=1%</td>
<td>19.5</td>
<td>$228,547</td>
<td>$214,795</td>
<td>$13,752</td>
</tr>
<tr>
<td>r=2%</td>
<td></td>
<td>$208,193</td>
<td>$193,882</td>
<td>$14,311</td>
</tr>
<tr>
<td>r=3%</td>
<td></td>
<td>$190,530</td>
<td>$176,152</td>
<td>$14,378</td>
</tr>
</tbody>
</table>

Source: authors Calculations.
* Expected Remaining Life, data are from Arias 2014.
Key: APV = Present Value of Life-Annunity, see Equation (1)
ACLE = Annuity Certain with ERL as the time horizon, see Equation (3)

These results are consistent with the Jensen Inequality theorem which predicts that ACLE overstates APV. Therefore, consistent with economic theory, if one accepts that APV is the correct way to evaluate life-annuities then one must conclude that models based on ACLE incorrect.

4. Conclusion

This paper shows that the two methods of evaluating Social Security Wealth (SSW), the present value of Social Security retirement benefits, lead to different results. When compared to the theoretically correct method of calculating annuity value as the expected present value, calculations using the discounting the cash flow through life expectancy method overstate value of SSW.

References


